OCR RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 1 5 7 4 0 6 1 3 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** You are given that $\mathrm{f}(x) = \dfrac{1}{(1-2x)^2}$.

Find $\mathrm{f}'(x)$, $\mathrm{f}''(x)$ and $\mathrm{f}'''(x)$. Hence obtain the Maclaurin series for $\mathrm{f}(x)$ as far as the term in x^3.

By considering the equivalent binomial expansion, give the set of values of x for which the Maclaurin series is valid. **[7]**

(b) A curve has polar equation $r = a\sin 3\theta$, where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(i) Sketch the curve. **[2]**

(ii) Find, in terms of a, the cartesian coordinates of the point on the curve furthest from the origin. **[4]**

(iii) Find, in terms of a, the area of the region enclosed by the curve. **[5]**

2 **(a)** **(i)** Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$ **[3]**

(ii) Given that $\cos 5\theta = 0$ but $\cos\theta \neq 0$, find in surd form the two possible values for $\cos^2\theta$.

Hence show that $\cos 18° = \left(\dfrac{5+\sqrt{5}}{8}\right)^{\frac{1}{2}}$.

Find, in similar form, an expression for $\sin 18°$. **[6]**

(b) **(i)** Find the cube roots of the complex number $4(\sqrt{3}+\mathrm{j})$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $0 < \theta < 2\pi$. Illustrate the roots on an Argand diagram. **[7]**

The points representing the two roots with smallest values of θ are P and Q. The mid-point of PQ is M, and M represents the complex number w.

(ii) Find the argument of w. Write down the smallest positive integer n for which w^n is a real number. **[2]**

3 You are given the matrix $\mathbf{A} = \begin{pmatrix} k & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix}$.

(i) Show that when $k = 5$ the determinant of **A** is zero. Obtain an expression for the inverse of **A** when $k \neq 5$. **[7]**

(ii) Solve the matrix equation

$$\begin{pmatrix} 4 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix},$$

giving your answer in terms of p. **[5]**

(iii) Find the value of p for which the matrix equation

$$\begin{pmatrix} 5 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix}$$

has a solution. Give the general solution in this case and describe it geometrically. **[6]**

Section B (18 marks)

4 **(i)** Prove, using exponential functions, that $\cosh^2 u - \sinh^2 u = 1$. **[2]**

(ii) Given that $y = \operatorname{arsinh} x$, show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}},$$

and that

$$y = \ln\left(x + \sqrt{1+x^2}\right).$$ **[9]**

(iii) Show that

$$\int_0^2 \frac{1}{\sqrt{4+9x^2}}\,dx = \tfrac{1}{3}\ln\left(3+\sqrt{10}\right).$$ **[4]**

(iv) Find, in exact logarithmic form,

$$\int_0^1 \frac{1}{\sqrt{1+x^2}} \operatorname{arsinh} x \,dx.$$ **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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4756/01 Further Methods for Advanced Mathematics (FP2)

PRINTED ANSWER BOOK

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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (54 marks)

1 (a)	

1 (b) (i)	
1 (b) (ii)	

1 (b) (iii)	

2 (a) (i)	

2 (a) (ii)	
2 (b) (i)	
	(answer space continued on next page)

2 (b) (i)	**(continued)**
2 (b) (ii)	

3 (i)	

3 (ii)	
	(answer space continued on next page)

3 (ii)	(continued)
3 (iii)	
	(answer space continued on next page)

3 (iii)	(continued)

Section B (18 marks)

4 (i)	
4 (ii)	
	(answer space continued on next page)

4 (ii)	(continued)

4 (iii)	

4 (iv)	

PLEASE DO NOT WRITE ON THIS PAGE

GCE

Mathematics (MEI)

Advanced GCE

Unit **4756:** Further Methods for Advanced Mathematics

Mark Scheme for June 2013

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

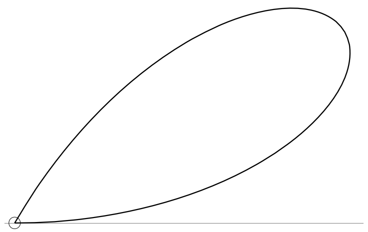
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

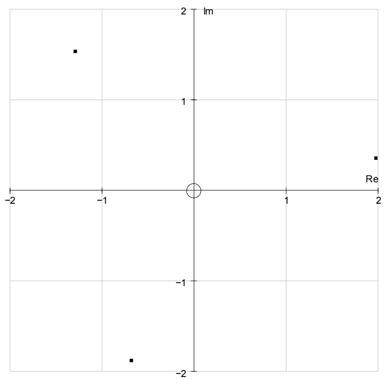
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(a)		$f(x)=(1-2x)^{-2}$			
			$\Rightarrow f'(x)=-2(1-2x)^{-3}\times -2=4(1-2x)^{-3}$	M1 A1	Derivative in the form $k(1-2x)^{-3}$ o.e. Any correct form www	For first derivative
			$\Rightarrow f''(x)=24(1-2x)^{-4}$	A1	Any correct form www	
			$\Rightarrow f'''(x)=192(1-2x)^{-5}$	A1	Any correct form www	
			$\Rightarrow f(0)=1, f'(0)=4,$ $f''(0)=24, f'''(0)=192$			
			$\Rightarrow f(x)=1+4x+\frac{24}{2!}x^2+\frac{192}{3!}x^3+\ldots$	M1	Using Maclaurin series with derivatives evaluated at $x=0$	Must have $r!$ in denominator
			$\Rightarrow f(x)=1+4x+12x^2+32x^3+\ldots$	A1		SR: after M0M0 B2 for correct binomial
			Valid for $-1<2x<1$ $\Rightarrow -\frac{1}{2}<x<\frac{1}{2}$	B1	Strict inequalities	
				[7]		
1	(b)	(i)		B2 **[2]**	For a complete loop correct at the origin and at the extremity	Ignore beyond $0\le\theta\le\pi/3$. Incomplete loop B0. Give B1 for wrong shape at one of origin or extremity
1	(b)	(ii)	$\theta=\frac{\pi}{6}$	B1	s.o.i.	
			$r=a$	B1	s.o.i.	
			$\Rightarrow x=a\cos\frac{\pi}{6}=\frac{a\sqrt{3}}{2}$	M1	Using $x=r\cos\theta$ and $y=r\sin\theta$ with a value of θ	
			and $y=a\sin\frac{\pi}{6}=\frac{a}{2}$	A1	Both. Condone $0.87a$	
				[4]		

Question			Answer	Marks	Guidance	
1	(b)	(iii)	$A=\int_0^{\frac{\pi}{3}}\frac{1}{2}a^2\sin^2 3\theta\,\mathrm{d}\theta$	M1 A1	An integral expression including $\sin^2 3\theta$ Correct integral expression with limits	 Limits may be inserted below
			$=\frac{1}{4}a^2\int_0^{\frac{\pi}{3}}1-\cos 6\theta\,\mathrm{d}\theta$	M1	Using $\sin^2 3\theta=\frac{1}{2}-\frac{1}{2}\cos 6\theta$ and attempting integration. Dep. on 1st M1	Allow sign and factor errors, but must be $\cos 6\theta$
			$=\frac{1}{4}a^2\left[\theta-\frac{1}{6}\sin 6\theta\right]_0^{\frac{\pi}{3}}$	A1	Correct result of integration	i.e. $\int\sin^2 3\theta\,\mathrm{d}\theta=\frac{1}{2}\theta-\frac{1}{12}\sin 6\theta$
			$=\frac{1}{12}\pi a^2$	A1 **[5]**	Dependent on previous A1	Allow awrt $0.26a^2$

Question			Answer	Marks	Guidance	
2	(a)	(i)	$\cos 5\theta + \text{j}\sin 5\theta = (\cos\theta + \text{j}\sin\theta)^5$			
			$= c^5 + 5c^4\text{j}s + 10c^3\text{j}^2s^2 + 10c^2\text{j}^3s^3 + 5c\text{j}^4s^4 + \text{j}^5s^5$	M1	Expanding $(c+\text{j}s)^5$ (real terms only)	Allow one error. Must get beyond $^5C_2$. Must collect terms
			$= c^5 - 10c^3s^2 + 5cs^4 + \text{j}(5c^4s - 10c^2s^3 + s^5)$			
			$\Rightarrow \cos 5\theta = c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2$	M1	Separating real part and replacing s^2 with $1-c^2$	Independent of M1
			$= c^5 - 10c^3 + 10c^5 + 5c(1 - 2c^2 + c^4)$			
			$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	A1(ag) **[3]**	Completion www in real part	
2	(a)	(ii)	$\theta = 18^\circ \Rightarrow \cos 5\theta = 0$ *			
			$\Rightarrow 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$			
			$\cos\theta \neq 0 \Rightarrow 16\cos^4\theta - 20\cos^2\theta + 5 = 0$	B1	This equation s.o.i.	
			$\Rightarrow \cos^2\theta = \dfrac{20 \pm \sqrt{20^2 - 4\times16\times5}}{2\times16}$	M1 A1	Solving a 3-term quadratic Unsimplified values of $\cos^2\theta$	Allow one error
			$\Rightarrow \cos\theta = \pm\left(\dfrac{5+\sqrt{5}}{8}\right)^{\frac{1}{2}}$ or $\pm\left(\dfrac{5-\sqrt{5}}{8}\right)^{\frac{1}{2}}$			SC Answers unsupported www B1
			$\cos 18^\circ$ is closest to 1 $\Rightarrow \cos 18^\circ = \left(\dfrac{5+\sqrt{5}}{8}\right)^{\frac{1}{2}}$	A1(ag)	Justifying selection of this root	To include *
			$\cos^2 18^\circ + \sin^2 18^\circ = 1$	M1	Using $\cos^2\theta + \sin^2\theta = 1$	
			$\Rightarrow \dfrac{5+\sqrt{5}}{8} + \sin^2 18^\circ = 1$			
			$\Rightarrow \sin^2 18^\circ = \dfrac{3-\sqrt{5}}{8}$ and $\sin 18^\circ > 0$			
			$\Rightarrow \sin 18^\circ = \left(\dfrac{3-\sqrt{5}}{8}\right)^{\frac{1}{2}}$	A1 **[6]**	Must have this form	

Question			Answer	Marks	Guidance	
2	**(b)**	**(i)**	$4\sqrt{3}+4\text{j}=8e^{\text{j}\frac{\pi}{6}}$	B1B1	$8,\ \dfrac{\pi}{6}$	Condone decimal equivalents for arguments throughout (to 2 s.f.).
			Cube roots are $re^{\text{j}\theta}$			
			$r^3=8\Rightarrow r=2$	B1ft	$\sqrt[3]{\text{their } 8}$	
			$3\theta=\dfrac{\pi}{6}\Rightarrow\theta=\dfrac{\pi}{18}$	B1ft	$\dfrac{1}{3}$ of their $\dfrac{\pi}{6}$	Radians only
			$\pm\dfrac{2\pi}{3}$	M1		
			$\Rightarrow\quad\theta=\dfrac{\pi}{18},\dfrac{13\pi}{18},\dfrac{25\pi}{18}$	A1	Accept $-\dfrac{11\pi}{18}$	Radians only
			 	B1 **[7]**	Approx. order 3 rotational symmetry. 1st root in $0<\arg z<\pi/4$ 2nd root in 2nd quadrant 3rd root in $5\pi/4<\arg z<3\pi/2$	Ignore numbers etc. on diagram
2	**(b)**	**(ii)**	$\arg w=\dfrac{1}{2}\left(\dfrac{\pi}{18}+\dfrac{13\pi}{18}\right)=\dfrac{7\pi}{18}$	B1		
			$n=18$	B1 **[2]**		

Question			Answer	Marks	Guidance	
3	(i)		$\det(\mathbf{A}) = k(4+9)+7(-4-3)+4(-6+2)$ $=13k-65$ $\Rightarrow$ no inverse if $k = 5$ $\mathbf{A}^{-1} = \frac{1}{13k-65}\begin{pmatrix} 13 & -26 & -13 \\ 7 & -2k-4 & -3k+8 \\ -4 & 3k-7 & -2k+14 \end{pmatrix}$	M1A1 B1(ag) M1 A1 M1 A1 **[7]**	Obtaining $\det(\mathbf{A})$ in terms of k May be verified separately At least 4 cofactors correct (including one involving k) Six signed cofactors correct Transposing and ÷ by $\det(\mathbf{A})$. Dependent on previous M1M1	Allow one error. isw unsimplified 65 − 13k M1A0 and allows B1 below M0 if more than 1 is multiplied by the corresponding element Mark final answer
3	(ii)		When $k = 4$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-13}\begin{pmatrix} 13 & -26 & -13 \\ 7 & -12 & -4 \\ -4 & 5 & 6 \end{pmatrix}\begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix}$	M1 M2	Substituting $k = 4$ Correct use of inverse	One correct element. Condone missing determinant. M0 if wrong order
			OR e.g. $\begin{matrix} 6x-13y=p+4 \\ 4x-13y=3p-4 \end{matrix} \Rightarrow x=-p+4$ M2 M1		Eliminating one unknown in two different ways and reaching one unknown in terms of p Finding the other two unknowns	
			$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-13}\begin{pmatrix} 13p-52 \\ 7p-20 \\ -4p+17 \end{pmatrix}$	A2 **[5]**	Dependent on all M marks. Terms must be collected. Give A1 for one correct	$x=-p+4$, $y=-\frac{7}{13}p+\frac{20}{13}$, $z=\frac{4}{13}p-\frac{17}{13}$ λ × correct vector (λ ≠ 0) A1

Question			Answer	Marks	Guidance	
3	(iii)		e.g. $7x-13y=p+4, 7x-13y=3p-4$ (or $4x+13z=7-2p, 4x+13z=-1$) (or $8y+14z=p-10, 4y+7z=-3$) For solutions, $p+4=3p-4$ $\Rightarrow \quad p=4$	M2 A1	Eliminating one unknown in two different ways & obtaining a value of p	Or $7x-13y=8$ Or $8x+26z=3p-14$ Or $4y+7z=5-2p$
			OR M2 $p=4$ A1		A method leading to an equation from which p could be found	E.g. setting $z=0$, augmented matrix, adjoint matrix, etc.
			$x=\lambda,\ y=\frac{7}{13}\lambda-\frac{8}{13},\ z=-\frac{4}{13}\lambda-\frac{1}{13}$ Straight line	M1 A1 B1 **[6]**	Obtaining general soln. by e.g. setting one unknown = λ and finding equations involving the other two and λ Any correct form Accept "sheaf", "pages of a book", etc.	Accept unknown instead of λ $x=\frac{13}{7}\lambda+\frac{8}{7},\ y=\lambda,\ z=-\frac{4}{7}\lambda-\frac{3}{7}$ $x=-\frac{13}{4}\lambda-\frac{1}{4},\ y=-\frac{7}{4}\lambda-\frac{3}{4},\ z=\lambda$ Independent of all previous marks. Ignore other comments

Question			Answer	Marks	Guidance	
4	(i)		$\cosh u = \frac{e^u + e^{-u}}{2} \Rightarrow \cosh^2 u = \frac{e^{2u} + 2 + e^{-2u}}{4}$			
			$\sinh u = \frac{e^u - e^{-u}}{2} \Rightarrow \sinh^2 u = \frac{e^{2u} - 2 + e^{-2u}}{4}$	B1	Numerators of both expressions	Accept other variables
			$\Rightarrow \cosh^2 u - \sinh^2 u = 1$	B1(ag)	Completion www	
			OR $\cosh u + \sinh u = e^u$			
			$\cosh u - \sinh u = e^{-u}$			
			$\Rightarrow \cosh^2 u - \sinh^2 u = e^u \times e^{-u}$ B1		Both expressions s.o.i. and multiplication	
			$\Rightarrow \cosh^2 u - \sinh^2 u = 1$ B1(ag)		Completion www	
				[2]		
4	(ii)		$y = \text{arsinh}\, x \Rightarrow x = \sinh y$			
			$\Rightarrow \frac{dx}{dy} = \cosh y$	M1	$\sinh y = \ldots$ and differentiating w.r.t. y or x	Or $\cosh y \frac{dy}{dx} = 1$ or differentiating (*)
			$\Rightarrow \frac{dy}{dx} = \frac{1}{\cosh y}$	A1	o.e.	
			$\Rightarrow \frac{dy}{dx} = (\pm)\frac{1}{\sqrt{1+\sinh^2 y}} = (\pm)\frac{1}{\sqrt{1+x^2}}$	A1(ag)	Completion www with valid intermediate step	$\frac{dy}{dx} = \pm\frac{1}{\sqrt{1+x^2}}$ as final answer or $\pm$ not considered scores max. 3/4
			y is an increasing function so take + sign	B1	Validly rejecting negative value	Or $\cosh y \geq 1$, or $\cosh y > 0$
			$x = \sinh y \Rightarrow x = \frac{e^y - e^{-y}}{2}$	B1	x in exponential form	
			$\Rightarrow e^y - e^{-y} = 2x$			
			$\Rightarrow e^{2y} - 2xe^y - 1 = 0$	M1	Obtaining quadratic in e^y	
			$\Rightarrow (e^y - x)^2 = 1 + x^2$			
			$\Rightarrow e^y = x \pm \sqrt{1+x^2}$	M1	Solving to reach e^y. Dep. on M1 above	Allow one slip
			$\Rightarrow y = \ln\left(x(\pm)\sqrt{1+x^2}\right)$ (*)	A1(ag)	Completion www	
			$x - \sqrt{1+x^2} < 0$ so take + sign	B1	Validly rejecting negative root	e.g. $e^y > 0$
				[9]		

Question			Answer	Marks	Guidance	
4	(iii)		$\int_0^2 \frac{1}{\sqrt{4+9x^2}}\,dx = \frac{1}{3}\int_0^2 \frac{1}{\sqrt{\frac{4}{9}+x^2}}dx$			
			$=\frac{1}{3}\left[\operatorname{arsinh}\frac{3x}{2}\right]_0^2$	M1 A1A1	Integral involving arsinh $\frac{1}{3}$, $\frac{3x}{2}$ o.e.	
			$=\frac{1}{3}\operatorname{arsinh}3$			
			OR $=\frac{1}{3}\left[\ln\left(x+\sqrt{x^2+\frac{4}{9}}\right)\right]_0^2$ M1 A1A1		Integral in form $\ln\left(kx+\sqrt{k^2x^2+...}\right)$ $\frac{1}{3}$, $x+\sqrt{x^2+\frac{4}{9}}$ or $3x+\sqrt{9x^2+4}$	Or $\frac{3x}{2}+\sqrt{\frac{9x^2}{4}+1}$
			OR $x=\frac{2}{3}\sinh u \Rightarrow \frac{dx}{du}=\frac{2}{3}\cosh u$ M1 A1 $\int_0^2 \frac{1}{\sqrt{4+9x^2}}\,dx = \int_0^{\ln(3+\sqrt{10})}\frac{1}{3}\,du$ A1		Using a sinh substitution Correct substitution $\int \frac{1}{3}\,du$	
			$=\frac{1}{3}\ln\left(3+\sqrt{10}\right)$	A1(ag) **[4]**	Completion with valid intermediate step(s)	Condone omitted brackets

Question			Answer	Marks	Guidance	
4	(iv)		$\int_0^1 \frac{1}{\sqrt{1+x^2}} \operatorname{arsinh} x \, dx$ $= \left[(\operatorname{arsinh} x)^2 \right]_0^1 - \int_0^1 \frac{1}{\sqrt{1+x^2}} \operatorname{arsinh} x \, dx$	M1	Parts with $u = \operatorname{arsinh} x$, $v' = \frac{1}{\sqrt{1+x^2}}$	Allow one error Allow equivalent form
			OR $\int \frac{1}{\sqrt{1+x^2}} \operatorname{arsinh} x \, dx = \int u \, du$ M1		Substitution with $u = \operatorname{arsinh} x$ or $x = \sinh u$	Must reach $\int u \, du$
			OR inspection M1		Recognising integrand as $k(\operatorname{arsinh} x)^2$	$k \neq 0$
			$\Rightarrow \frac{1}{2}(\operatorname{arsinh} x)^2$ $\Rightarrow I = \frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)\right)^2$	A1 A1 **[3]**	A correct indefinite integrand This answer only	 Mark final answer

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OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

Candidates performed very well on this question paper with a little over one-third of the entry scoring at least 60 marks and only about 5% scoring 20 marks or fewer. Question 1 (Maclaurin series, polar curves) was the best done question, followed by Question 3 (matrices), Question 4 (hyperbolic functions) and Question 2 (complex numbers).

A lot of the scripts were well presented and there seemed to be less use of supplementary sheets than in the past.

Candidates might have done even better if they had:

- read some of the questions more carefully, especially if they asked for more than one thing, such as Q3(iii) and Q4(ii);
- taken more care with 'elementary' algebra and numerical work, e.g. in Q3;
- written down more of their working, e.g. in Q3(iii) where it is inadvisable to do so much of the elimination 'in the head' and in Q4(iii) where a given answer had to be shown;
- taken more care with drawing diagrams, e.g. in Q1(b)(i) and Q2(b)(i);
- taken a little time to resolve the ambiguities of sign in Q4(ii): these are standard results and it is not inconceivable that they, or their close relatives, will reappear in a future series;
- improved their judgement in choosing an appropriate method for differentiation: it is much easier to use the Chain Rule in Q1(a) rather than the quotient rule;
- understood better what the inverse of a matrix does, i.e. that it can be used to solve systems of linear equations, such as those in Q3(ii); it was not necessary to start again from scratch;
- realised that parts labelled (i), (ii) etc. are linked, so Q3(i) was meant to be helpful in solving Q3(ii).

Comments on Individual Questions

1) **Maclaurin series, polar curves**

In part (a), candidates were asked to differentiate $\frac{1}{(1-2x)^2}$ repeatedly with respect to *x*, obtain the Maclaurin series, and give the domain of validity by considering the equivalent binomial expansion. The majority of candidates did all this as required, and very accurately. When differentiating, some used the quotient rule, which led to much unpleasantness if expressions were not simplified. Multiplying out the brackets was seen occasionally. A substantial number, having obtained the Maclaurin series, also obtained the binomial expansion, which was not always the same. The domain of validity was often correct but was sometimes omitted or not strict.

Part (b) was about the polar curve $r = a\sin 3\theta$. The sketches in (i) were often correct; a few candidates had a pointed extreme at $\theta = \frac{\pi}{6}$ or an incorrect form at the origin. Drawing further loops (beyond $0 \le \theta \le \frac{\pi}{3}$) was not penalised. The unfamiliar nature of (ii) put off some candidates: what was expected was that they would use their knowledge of the sine function and/or their sketches to conclude that the maximum value of *r* would occur when

$\theta = \frac{\pi}{6}$. Many did this, but some confused *x* and *y*. Others deployed all the formulae to do with cartesian and polar coordinates that they knew in futile attempts to reach an answer. Differentiating the given polar equation with respect to *θ* was fairly common. The area of the loop in (iii) was frequently correct: the overwhelming majority of candidates knew exactly what to do and had good ideas about how to do it, although sign and factor errors in the trigonometric identity and the integration were fairly common.

2) **Complex numbers**

Part (a) first asked candidates to produce a given expression for cos 5*θ* in terms of powers of cos *θ*. The technique was well known and usually carried out extremely accurately. The examiners ignored work which led to imaginary terms in the expansion of $(\cos\theta + j\sin\theta)^5$; had we not, maybe a couple more marks would have been lost. A small number of candidates went into $\left(z + \frac{1}{z}\right)^5$ mode: this can produce the required answer, but extremely few gave a complete correct argument by this method. Then candidates were asked to find the two possible values for $\cos^2\theta$ given that cos 5*θ* = 0 and cos *θ* ≠ 0, and go on to produce a given expression for cos 18°, and find a similar one for sin 18°. Most saw that they could use their quintic expression from part (i) to derive a quadratic equation in $\cos^2\theta$, which they solved generally accurately give or take a number of careless errors in applying the quadratic formula, and scored the first three marks. The fourth mark, for showing the given expression was cos 18°, was awarded very infrequently; most were content to ignore where the 18° had come from, while very, very few considered other possibilities such as 54°. Some used their calculators to find the inverse cosine of the given expression. For sin 18°, a substantial number of candidates went back to the imaginary parts in (i): again, this can produce a correct answer (via a quintic in sin *θ*, one of whose roots is 1 and which has a repeated quadratic factor) but this was never seen, and fortunately most attempts involved the use of $\cos^2\theta + \sin^2\theta = 1$. The final answer was expected to be given "in similar form", i.e. simplified.

Part (b) (i), requiring the cube roots of $4\left(\sqrt{3} + j\right)$ to be obtained and plotted on an Argand diagram, met with widespread approval and many fully correct answers were seen. Errors, where they occurred, usually involved taking the modulus of $4\left(\sqrt{3} + j\right)$ to be 4 or 7. Most candidates knew that the cube roots occurred every $\frac{2\pi}{3}$ but the Argand diagram sometimes stretched the definition of rotational symmetry.

Part (ii) was less well done, with some candidates taking the wrong two points; the question stated that values of arguments in part (i) should be in the interval $0 < \theta < 2\pi$. *n* was sometimes not an integer and was less frequently correct than the argument.

3) **Matrices and linear equations**

Finding the inverse of a 3 × 3 matrix is a familiar process for almost all candidates, and nearly all the marks lost in part (i) were as a result of arithmetical or simple algebraic slips, for example, $13k - 52 - 13 = 13k - 39$. One or two multiplied their cofactors by the elements of the original matrix. A variety of methods were employed for finding the determinant, including Sarrus' method.
In part (ii), although most candidates realised that they could use the inverse matrix they had found in (i) with *k* = 4, many others started again with algebra, wasting much time and making many errors. Some who used the matrix 'lost' the determinant.

Part (iii) caused the most trouble. An efficient method to find *p* is to pick one of the unknowns, eliminate it in two different ways obtaining equations which are not independent, and then find *p*. Once again, this was much more accurately done when candidates wrote down organised working, rather than trying to do the manipulation in their heads. Also once again, there was much 'tail-chasing' as some candidates eliminated first *x*, then *y*, and then *z*, filling the whole answer space (and sometimes supplementary sheets) with futile algebra. A neat method is to observe that 2 × equation (2) + equation (3) gives $5x - 7y + 4z = 4$, so $p = 4$; this was very rarely seen. Having failed to find *p*, many candidates gave up and did not try to find the general solution: those who did sometimes found a factor of $\frac{1}{13}$ unappealing and multiplied everything by 13 to remove it, thereby changing the 'point' on the solution line as well as the 'direction'. The geometric description of the general solution was frequently correct but was sometimes omitted.

4) **Hyperbolic functions**

In part (i), a proof that $\cosh^2 u - \sinh^2 u = 1$ was wanted. Most candidates knew what to do, but this part was perhaps less well done than expected, with too many careless errors. e^{-2u} was liable to become e^{2u} or even u^{-2u} and there were various sign slips.

Part (ii) asked for the proofs of two given answers; the derivative with respect to *x* of arsinh *x*, and its logarithmic form. Both seemed familiar and very many candidates were able to score 7/9, losing the marks given for resolving the ambiguity of sign in both expressions. For the logarithmic form, many spurious arguments involving gradient, "ln cannot be negative", $\left(x+\sqrt{1+x^2}\right)\left(x-\sqrt{1+x^2}\right)=1$ (sic) and "principle (sic) value" were seen.

Part (iii), involving an arsinh integral, was very well done. The final mark was withheld from candidates who omitted essential working required to show the given answer, and there were many of these.

Part (iv) was a good discriminator. Parts was probably the most common method employed, but candidates often could not make progress beyond the 'first line', not realising that their integral expressions on both sides could be combined. Substitution of $u = \text{arsinh}\, x$ often ensured better progress and produced a correct indefinite integral. A few candidates recognised the standard form $\int f(x)f'(x)\,dx = \frac{1}{2}\left(f(x)\right)^2 + c$. Many candidates lost the final mark through sloppy use of brackets or the invention of new 'log laws' which, for instance, caused $\frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)\right)^2$ to become $\ln\left(1+\sqrt{2}\right)$.

Unit level raw mark and UMS grade boundaries June 2013 series

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0
4798/01	(FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241/01	(Z1) Statistics 1	Raw	72	55	48	41	35	29	0
		UMS	100	80	70	60	50	40	0
G242/01	(Z2) Statistics 2	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243/01	(Z3) Statistics 3	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0